PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
Independent Public School

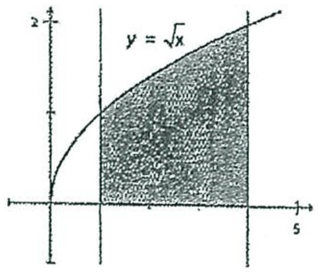
Methods 3 Integration Practice Test

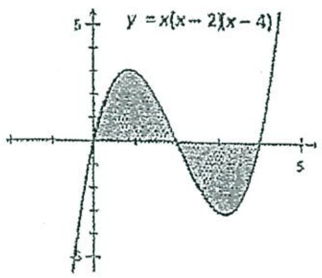
Calculator Assumed

Name________________________________

Total Mark____________________________

1. Determine the area of the shaded regions [7 marks]





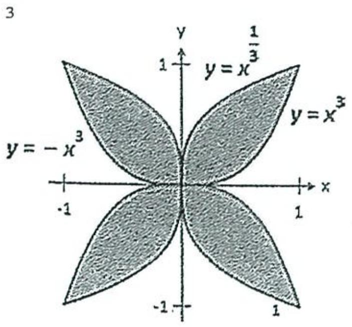
2. Use a calculus method to find the area of the region trapped by the curves [9 marks]

a) $y = x^2$ and $y = -x+2$

b) $y = 2x(x + 2)$ *and* $y = x + 2$

3. The accompanying diagram shows the logo of a particular company. The logo consists of four petals as shown.

All measurements are in metres. Find the total area of the petals. [3]



4. The instantaneous rate with which the temperature, θ degrees Celsius, of a body changes with respect to time, t minutes, is modelled by $\frac{d\theta}{dt}=2-0.05t$.

(a) Find when the maximum temperature of the body occurred.

(b) Find the net change in temperature in the (i) first 5 minutes (ii) 5th minute.

5. The marginal cost for producing x items of a product is given by $dC/dx = 0.04x$, where \$C is the cost of producing x items of a product.

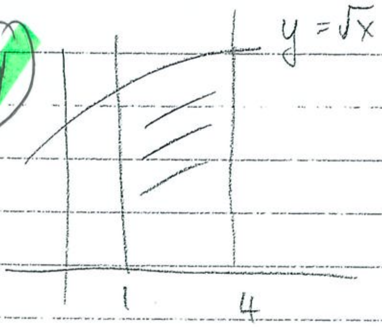
(a) Given that the fixed cost is \$20, find the cost of producing 100 of these items.

(b) Find the net change in cost if the number of items produced is changed from 100 to 200.

INTEGRATION PRACTICE TEST 2

SOLUTIONS

1



a)

$$\int_1^4 \sqrt{x}\, dx = \frac{x^{3/2}}{3/2}\Bigg|_1^4$$

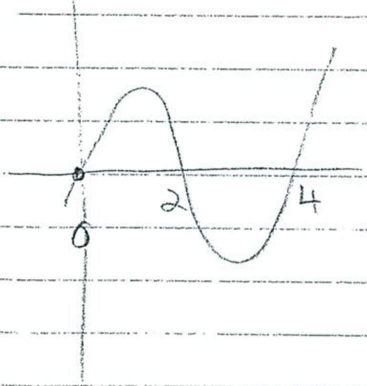
$$= \frac{2x^{3/2}}{3}\Bigg|_1^4$$

$$= \left(\frac{2(2^3)}{3}\right) - 2\left(\frac{1}{3}\right)$$

$$= \frac{16}{3} - \frac{2}{3}$$

$$= \frac{14}{3} \text{ units}^2$$

b)



$$\int_0^2 x(x-2)(x-4) + \int_2^4 \left|\left(x(x-2)(x-4)\right)\right|$$

$$= \int_0^2 x^3 - 6x^2 - 8x + \int_2^4 \left| x^3 - 6x^2 - 8x \right|$$

$$= \left(\frac{x^4}{4} - 2x^3 + 4x^2\right)\Bigg|_0^2 + \left|\left(\frac{x^4}{4} - 2x^3 + 4x^2\right)\Bigg|_2^4\right|$$

$$= \left(\frac{16}{4} - 16 + 16\right) + \left|(64 - 128 + 64) - \left(\frac{16}{4}\right)\right|$$

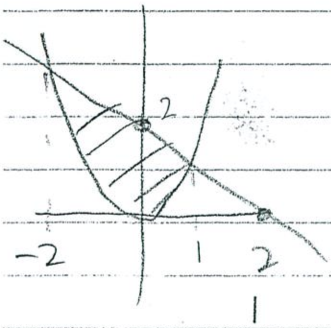
$$= (4) + |-4|$$

$$= 8 \text{ units}^2$$

2

(9 marks)

a) $y = x^2 \qquad y = -x + 2$



$-x + 2 = x^2$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$\therefore x = -2, 1$ ✓

$$\int_{-2}^{1} -x + 2 - x^2 \, dx = -\frac{x^2}{2} + 2x - \frac{x^3}{3} \Big|_{-2}^{1}$$

$$= \left(-\tfrac{1}{2} + 2 - \tfrac{1}{3}\right) - \left(-2 - 4 + \tfrac{8}{3}\right)$$ ✓

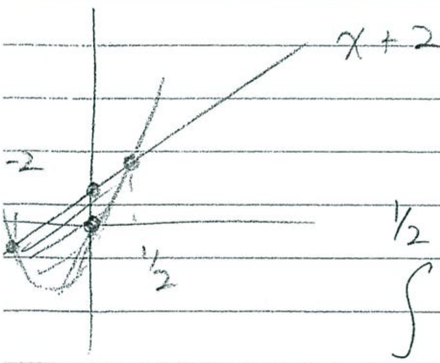
$$= \left(1\tfrac{1}{6}\right) - \left(-3\tfrac{1}{3}\right)$$

$$= 1\tfrac{1}{6} + 3\tfrac{2}{6}$$

$$= \underline{4\tfrac{1}{2} \text{ units}^2}$$ ✓

b)

$y = 2x^2 + 4x \qquad y = x + 2$



$2x^2 + 4x = x + 2$ ✓

$2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$\therefore x = \tfrac{1}{2} \quad x = -2$

$$\int_{-2}^{1/2} (x+2) - (2x^2 + 4x) = \int_{-2}^{1/2} -3x + 2 - 2x^2$$ ✓

$$= -\frac{3}{2}x^2 + 2x - \frac{2}{3}x^3 \Big|_{-2}^{1/2}$$ ✓

$$= \left(-\tfrac{3}{8} + 1 - \tfrac{1}{12}\right) - \left(-6 - 4 + \tfrac{16}{3}\right)$$

$$= \left(\frac{-9 + 24 - 2}{24}\right) - \left(-\frac{14}{3}\right)$$ ✓

$$= \frac{13}{24} + \frac{14}{3}$$ ✓

$$= \frac{13}{24} + \frac{112}{24} = \frac{125}{24} \text{ units}^2$$

3

Petals

$$\int_0^1 x^{1/3} - x^3\,dx = \left.\frac{x^{4/3}}{4/3} - \frac{x^4}{4}\right|_0^1 \checkmark$$

$$= \left(\frac{3}{4} - \frac{1}{4}\right) \checkmark$$

$= 2/4$ $1/2\ u^2$ for each petal

$\therefore$ Total area $2u^2$ ✓

4

$$\frac{d\theta}{dt} = 2 - 0.05t$$ $\cdot 05 = \frac{5}{100} = \frac{1}{20}$ ✓

$$\theta = 2t - \frac{0.05t^2}{2}$$

a) $\frac{d\theta}{dt}$ = Max

$2 = 0.05t$

$\underline{t = 40}$ ✓

b) $$\int_0^5 2 - 0.05t = \left.\left(2t - \frac{0.05t^2}{2}\right)\right|_0^5$$ $\cdot 05 \times 25 = \cdot 125$ ✓

$$= \left(10 - \frac{\cdot 05(5^2)}{2}\right) \checkmark$$

$$= \left(10 - \frac{1.25}{2}\right)$$

$= 10 - 0.625 = 9.375$ ✓

$$\int_4^5 2 - 0.05t = (9.375) - \left(8 - \frac{\cdot 05(16)}{2}\right) \checkmark$$

$= 9.375 - 7.600$ ✓

$= \underline{1.775}$ ✓

5

$\frac{dC}{dx} = 0 \cdot 04x$

a) $C = 0.02x^2 + 20$

$\therefore \quad C(100) = 0.02(100)^2 + 20$

$= \$220$

b) $\int_{100}^{200} 0.04x \, dx = 0.02x^2 \Big|_{100}^{200}$

$= 0.02(40000) - 200$

$= \$600$